CALGARY JUNIOR HIGH TEAM MATH ATTACK - PART B

Saturday, December 9th, 2017

Sponsored by the University of Calgary Department of Mathematics and Statistics

Time: 30 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.
Students are not allowed to leave the room during testing time.

Instructions

1. Do not open the contest paper until you are told to do so.
2. You may use rulers, compasses and paper for rough work, but all problems can be solved without additional aid.
3. Write your team name, member names, and school at the top of the response sheet. Print clearly. You will submit the response sheet for marking, but you may keep the problem booklet.
4. This is mostly a short answer test. A blank space to place your answer follows each question on the response sheet. To receive full marks, you must simply write your numerical answer in the blank space. For problems that require written solutions (Part 2: #7, Part 3: #6), be sure to be clear and concise in your solution to receive full marks. No partial marks will be given.
5. Scoring: Each correct answer in Part 1 is worth 1 point. Each correct answer in Part 2 is worth 2 points. Each correct answer in Part 3 is worth 3 points. There is no penalty for an incorrect answer.
6. Tiebreakers: In the event that two teams have the same number of total points, the team that correctly answered more questions wins. In the event two teams have the same number of points and correctly answered the same number of questions, the team that misses the earlier question loses the tiebreaker.
7. If there are words in italics at the end of the question, it is the source that the problem is drawn from. It does not change the question in any way.
8. When your supervisor tells you to begin, you will have 30 minutes of working time.

Good luck!

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PART B PROBLEMS

Part 1 - Easy

6. ABCD is a positive four-digit number. A = D, B = 4A, and C = B - 1. What is the value of (10A + B)(10C + D) + 1 if it is a prime number?

7. In a group of five friends:
• Amy is taller than Carla.
• Dan is shorter than Eric but taller than Bob.
• Eric is shorter than Carla.
Who is the shortest?

8. Let $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$. What is a?

9. A square is inscribed in a circle with an area of 81π. What is the area of the square?

10. What is the units digit of 14^{2017} ?

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PART B PROBLEMS

Part 2 - Medium

6. Bobby and Amir run for 45 minutes on a circular track. Bobby runs clockwise at 250 m / min and uses the inner lane with radius 50 m. Amir runs counterclockwise at 300 m / min and uses the outer lane with a radius of 60 m, starting at the same place as Bobby. How many times after they start running do they pass each other?

7. Show that this is flawed reasoning:

$$x = y$$
$$\therefore x^2 = xy$$
$$\Rightarrow x^2 - y^2 = xy - y^2$$
$$x + y = y$$
$$\because x = y, \therefore 2y = y.$$
$$\therefore 2 = 1$$
$$\Rightarrow 1 = 0.$$

8. Rectangle ABCD has AB = 10 and BC = 5. Points X and Y are outside of the rectangle with XA = YC = 6 and XB = YD = 8. Determine the length of XY.

9. Find the largest five-digit integer which is divisible by the sum of its digits.

10. Jane would take 9 hours to build a tower alone, and Ben would take 10 hours to build it alone. When they work together, they talk a lot, and their combined output decreases by 10 bricks per hour. Working together, they build the tower in 5 hours. How many bricks are in the tower?

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PART B PROBLEMS

Part 3 - Hard

6. Prove that $n^4 \leq 4^n$ for all positive integers n greater than 3.

7. Determine $3x_4 + 2x_5$ if x_1, x_2, x_3, x_4, x_5 satisfy the system of equations below.
(1986 AIME #4)

$$2x_1 + x_2 + x_3 + x_4 + x_5 = 6$$
$$x_1 + 2x_2 + x_3 + x_4 + x_5 = 12$$
$$x_1 + x_2 + 2x_3 + x_4 + x_5 = 24$$
$$x_1 + x_2 + x_3 + 2x_4 + x_5 = 48$$
$$x_1 + x_2 + x_3 + x_4 + 2x_5 = 96.$$

8. Let X be the set of all binary integers that can be written using exactly 4 zeros and 9 ones, where leading zeros are allowed. If all possible subtractions (in which one element of X is subtracted from another) are performed, how many times is the answer 1 obtained?

9. A commonly used approximation of pi is 3.14. Find the least positive integer d such that if the area of the circle with diameter d is calculated using the approximation 3.14, the error will exceed 1.

10. If non-negative integers *a, b, c, d* satisfy the equation $a + b + c + d = 40$, how many possible (a, b, c, d) solutions exist?